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Proposal for preparing entangled coherent states using atom-cavity-mode Raman interaction

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Abstract. A scheme for preparing entangled coherent states is presented. It is based on the atom-cavity-mode Raman interaction. We also generalize this method for generating multi-mode entangled coherent states. Finally, our experimental feasibility is discussed.

PACS. 03.65.Ud Entanglement and quantum nonlocality (e.g. EPR paradox, Bell's inequalities, GHZ states, etc.) – 03.67.Hk Quantum communication – 42.50.Dv Nonclassical field states; squeezed, antibunched, and sub-Poissonian states; operational definitions of the phase of the field; phase measurements

Entanglement is one of the most striking features of quantum mechanics. This term "entanglement" is used for states of composite system that cannot be separated into the product of states of subsystems. Entanglement implies the correlation between separated subsystems and indicates the phenomenon of quantum nonlocality. Hagley *et al.* and Turchette *et al.* [1] have all demonstrated schemes for preparing the Einstein-Podolsky-Rosen (EPR) states, which opens the way for new tests of nonlocality and makes it possible to realize quantum communication based atoms. In 1989, a conception of GHZ state is presented by Greenberger *et al.* [2]. After this, much attention is being paid to the quantum entanglement. The entanglement is a resource with which to perform quantum information processing tasks, such as quantum computing [3], quantum error correction [4], dense coding [5] and quantum teleportation [6–8]. According to multiparticle entanglement purification protocols, a scheme for establishing the entanglement between multiparticle located at different nodes of a communication network has be proposed [9]. In this paper, we further consider how to prepare the entangled coherent states (ECS). A proposal for preparing the ECS is presented. It is based on the atom-cavity mode Raman interaction. The method in our paper is only local measurement on atom, whereas this technology is used extensively in quantum state engineering, so our scheme may be realized in practice.

We first describe a model which is used in our schemes. Let us consider the degenerate Raman interaction of an atom with a single-mode cavity-field. The degenerate Raman interaction describes a degenerate Λ-type three-level atom, in which the energies of the two lower states $|g\rangle$ and $|e\rangle$ is equal, interacting with a single-mode cavity field. The schematic diagram of the degenerate Λ-type three-level atom interacting with a single-mode field is displayed in Figure 1. When the atomic transition frequency ω_a is highly detuned from the cavity frequency ω_c, *i.e.* $\Delta = \omega_\mathrm{a} - \omega_\mathrm{c}$ is large, the upper state $|f\rangle$ can be adiabatically eliminated. Under this condition, the effective Hamiltonian for such a system is given by [10]

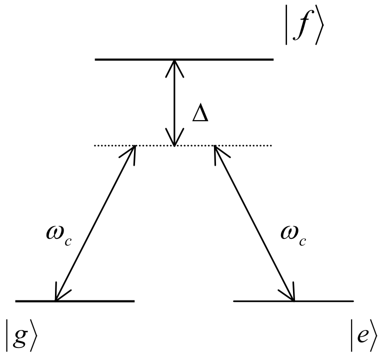

Fig. 1. Schematic diagram of the degenerate Λ-type three-level atom interacting with a single-mode field. ω_c is the cavity frequency, and Δ is the detuning.

$$H_{\mathrm{eff}} = -\lambda a^\dagger a(|e\rangle\langle g| + |g\rangle\langle e|) - a^\dagger a(\beta_1|g\rangle\langle g| + \beta_2|e\rangle\langle e|), \tag{1}$$

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where $\lambda = g_1 g_2/\Delta, \beta_1 = g_1^2/\Delta, \beta_2 = g_2^2/\Delta$, and g_1 (g_2) is the coupling constant of the transition between the states $|f\rangle$ and $|g\rangle$ $(|e\rangle)$, $a^\dagger$ and a are, respectively, the creation and annihilation operators for the cavity mode. For simplicity, we assume $g_1 = g_2 = g$, thus we have $\lambda = \beta_1 = \beta_2 = g^2/\Delta$. Further we assume that the cavity-mode may be prepared firstly in coherent state $|\alpha\rangle$ by driving the cavity with a classical current [11], and the atom is in the ground state $|g\rangle$, then the initial state of the whole system is $|\Psi(0)\rangle_{\mathrm{a-f}} = |\alpha\rangle \otimes |g\rangle$. In the interaction picture, the state vector of the system satisfies Schrödinger equation as follows

$$\mathrm{i}\frac{\mathrm{d}|\Psi(t)\rangle}{\mathrm{d}t} = H_{\mathrm{eff}}|\Psi(t)\rangle \tag{2}$$

the state vector of the system is given by

$$\begin{aligned} |\Psi(t=\tau)\rangle &= \exp(-\mathrm{i}H_{\mathrm{eff}}t)\,|\Psi(0)\rangle_{\mathrm{a-f}} \\ &= \frac{1}{2}\Bigg[\left(|\alpha\rangle + |\alpha \mathrm{e}^{2\mathrm{i}\left(g^2/\Delta\right)\tau}\rangle\right)|g\rangle \\ &\quad - \left(|\alpha\rangle - |\alpha \mathrm{e}^{2\mathrm{i}\left(g^2/\Delta\right)\tau}\rangle\right)|e\rangle\Bigg] \end{aligned} \tag{3}$$

where τ being the interaction time of the atom and the cavity-mode. We take $\tau = \pi\Delta/2g^2$ through adjusting the atomic velocity. Thus the state (3) can be rewritten as

$$|\Psi(t=\tau)\rangle = \frac{1}{2}[(|\alpha\rangle + |-\alpha\rangle)|g\rangle - (|\alpha\rangle - |-\alpha\rangle)|e\rangle]. \tag{4}$$

Secondly, we discuss how to prepare ECS. Recently, van Enk and Hirota [12] studied entangled nonorthogonal state in the context of bosonic entangled coherent state. The state is

$$\begin{aligned} |\Psi\rangle_{12} &= N_{12}\left(|\alpha\rangle_1|\alpha\rangle_2 - |-\alpha\rangle_1|-\alpha\rangle_2\right), \\ N_{12} &= \left[2 - 2\exp\left(-4|\alpha|^2\right)\right]^{-1/2}. \end{aligned} \tag{5}$$

Here $|\alpha\rangle_i$ $(i = 1, 2)$ labels ith coherent state mode. The ECS $|\Psi\rangle_{12}$ possess exactly one bit entanglement and the amount of entanglement is independent α except the limit case $|\alpha| \to 0$ [12]. In order for preparing the ECS, we need a Λ-type three-level atom with its state $|g\rangle$ and three coherent state field with amplitude $|\alpha|$. And assume that the Raman interaction occurs between the atom and the cavity modes, and further assume that their interaction time is all equal to $\pi\Delta/2g^2$. After interaction of the atom with mode 1, the state of the atom and mode 1 is described by equation (4). When the atom exits from the cavity 1, and enters cavity 2. After interaction, we have

$$\begin{aligned} |\Phi\rangle_{12} = \frac{1}{2}[&(|\alpha\rangle_1|\alpha\rangle_2 + |-\alpha\rangle_1|-\alpha\rangle_2)|g\rangle \\ &- (|\alpha\rangle_1|\alpha\rangle_2 - |-\alpha\rangle_1|-\alpha\rangle_2)|e\rangle]. \end{aligned} \tag{6}$$

If we let the atom interact with mode 3, the state of the whole system arrives at

$$\begin{aligned} |\Phi\rangle_{123} = \frac{1}{2}[&(|\alpha\rangle_1|\alpha\rangle_2|\alpha\rangle_3 + |-\alpha\rangle_1|-\alpha\rangle_2|-\alpha\rangle_3)|g\rangle \\ &- (|\alpha\rangle_1|\alpha\rangle_2|\alpha\rangle_3 - |-\alpha\rangle_1|-\alpha\rangle_2|-\alpha\rangle_3)|e\rangle]. \end{aligned} \tag{7}$$

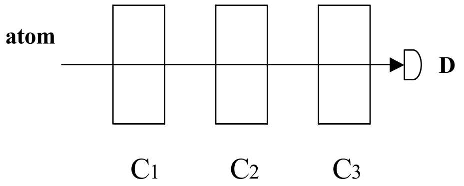

Fig. 2. The sketch of the preparation of entangled coherent states. C_i $(i = 1, 2, 3)$ represent cavity i. D refers to a detector.

In order to obtain ECS, a detection of the atom is need. If the atom is measured and found to be in state $|g\rangle$ or $|e\rangle$, for equations (6, 7), we have, respectively

$$|\Psi\rangle_{12} = N_{\pm}\left(|\alpha\rangle_1|\alpha\rangle_2 \pm |-\alpha\rangle_1|-\alpha\rangle_2\right), \tag{8}$$

and

$$|\Psi\rangle_{123} = N'_{\pm}\left(|\alpha\rangle_1|\alpha\rangle_2|\alpha\rangle_3 \pm |-\alpha\rangle_1|-\alpha\rangle_2|-\alpha\rangle_3\right) \tag{9}$$

where $N_{\pm}$ and $N'_{\pm}$ are normalized factors. The sketch of preparing entangled coherent states is displayed in Figure 2.

It is clear that we can generalize our results to the multimode case. A multimode ECS can be represented as

$$|\Psi\rangle_n = N_{\pm}^n\left(|\alpha\rangle_1|\alpha\rangle_2\cdots|\alpha\rangle_n \pm |-\alpha\rangle_1|-\alpha\rangle_2\cdots|-\alpha\rangle_n\right). \tag{10}$$

To see the fact that we need only n coherent state modes with amplitude $|\alpha|$ and a local measurement on the atom state.

Finally, let us give a discussion and conclusion. We have proposed scheme for generating entangled coherent state. These process of generation is achieved through the Raman interaction of atom with cavity-mode. In our scheme, only measurement on the atomic states are utilized. However, the relation of the cavity lifetime and the atomic state lifetime must be considered. It is well-known that there are some sensitive points which are typical in such experiments employing cavity QED phenomena such as the dissipative process due to cavity losses and atomic spontaneous emission, the dispersion in the atomic velocity, and the efficiency of atomic detection. However, if Rydberg atom of long lifetime and high-Q superconducting microwave cavities are considered, the atom-cavity-field interactions are supposed to dominate the dissipative processes. For Rydberg atom, the radiative time is $T_{\mathrm{atom}} = 3 \times 10^{-2}$ s [13], and the coupling constant g can take $2\pi \times 25$ kHz, we also take $\Delta = 100g$, this satisfies large detuning condition. Thus, the interaction time of atom with the cavity mode is $\tau = 10^{-3}$ s. The required time of preparing three cavity modes in entanglement is about 3×10^{-3} s, much shorter than T_{atom}. In fact, according to references [14,15], the atomic velocity can be achieved to $1\,000$ ms^{-1}, at this interval, the traveled distant is of the order of 10 m. On other hand, Because of the cavities lifetime $T_{\mathrm{cav}} = Q/(2\pi\nu_{\mathrm{a}})$, in terms of reference [15], we take: $\nu_{\mathrm{a}} = \omega_{\mathrm{a}}/2\pi = 50$ GHz, we then have $T_{\mathrm{cav}} \sim 10^{-12}Q$. We choose $Q > 10^{11}$ so that $T_{\mathrm{cav}} > 0.1$ s, much longer than the required time. Cavity with such as

Q factor have been reported in reference [16]. Thus, by using Rydberg atom of long lifetime and by choosing superconducting microwave cavities with an enough high-Q, there is sufficient time to achieve our generation of entangled states and teleportation experiment. We expect that our method may be used in quantum communication.

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